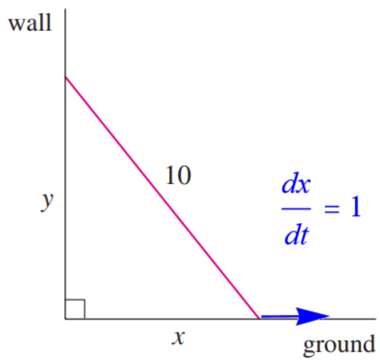
Exercise 34

According to the model we used to solve Example 2, what happens as the top of the ladder approaches the ground? Is the model appropriate for small values of y?

Solution

In Example 2 there's a 10-foot ladder that's leaning against a wall and sliding on the ground at a rate of 1 ft/s.



Since we want to know dy/dt when x is some constant, start with the Pythagorean theorem.

$$10^2 = x^2 + y^2$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\frac{d}{dt}(10^2) = \frac{d}{dt}(x^2 + y^2)$$

$$0 = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$0 = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$= x(1) + y\frac{dy}{dt}$$

$$= x + y\frac{dy}{dt}$$

Solve for dy/dt.

$$\frac{dy}{dt} = -\frac{x}{y}$$

Notice that

$$\lim_{y\to 0}\frac{dy}{dt} = \lim_{y\to 0} -\frac{x}{y} = -\infty,$$

which means the top of the ladder hits the floor with infinite speed. The model is clearly inappropriate for small values of y.